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Comparing Categorical Data Arrays across Opportunities: Adapting STATIS to CBA with Plithogenic Fuzzy Soft Sets

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Abstract: A method is proposed that adapts the STATIS methodology to correspondence analysis (CA), which allows analyzing data structures of two categorical variables on several occasions. On each occasion, a CA is applied and then a comparative analysis is performed between occasions in order to evaluate changes in the association between variables. The procedure allows: (1) Analyzing the variability of individual profiles on each occasion, projecting them onto spaces of optimal fit; (2) Comparing data structures between occasions, using a Hilbert - Schmidt distance, constructed as a direct function of the association between the variables that identify each occasion and inverse function of the association between the variables measured between occasions; (3) Representing individuals in a compromise space, using a score that captures the positions of their coordinates throughout the occasions; and (4) Performing the trajectory analysis of individuals throughout the occasions on the compromise space. The proposed method was applied in a nutritional assessment study of schoolchildren in 2017 and 2018, who were provided with food supplements. The results allowed us to identify trajectories of changes recorded in the children, facilitating the diagnosis of this problem. Additionally, an analysis based on plithogenic fuzzy soft sets was incorporated to model the uncertainty inherent in external factors, enriching the interpretation of nutritional trajectories.

Keywords: STATIS, AC, Hilbert -Schmidt, Plithogenic Fuzzy Soft Sets, Neutrosophy, Uncertainty.

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1 Introduction

STATIS is a methodology introduced by L'Hermier des Plantes [1] and developed by Christine Lavit [2], designed for the simultaneous analysis of multiple quantitative data matrices. The term STATIS is the French acronym for *Structuration de Tableaux A Trois Indices de la Statistique.* The methodology aims to simultaneously analyze data matrices corresponding to K instants. A presentation of the theoretical and practical aspects of this methodology can be found in Dazy & Le Barzic [3].

The fundamental questions addressed by STATIS include: 1) How do correlation structures between variables change across occasions? 2) Which variables are responsible for these changes? 3) How does each individual behave across occasions? These questions become especially relevant when working with categorical data subject to uncertainty and imprecision, particularly in longitudinal studies where unobserved factors can affect measurements.

The procedure follows a step-by-step strategy called ICI (Interstructure-Compromise-Intrastructure). In the initial phase, each data matrix is represented by a point in a multidimensional space, allowing for the identification of similarities and differences. Subsequently, a compromise matrix is constructed that synthesizes the structures from all instances, serving as a reference for comparisons. Finally, individual trajectories are projected onto this compromise space, visualizing the temporal evolution of the phenomena studied.

In this work, we adapt this methodology to the analysis of qualitative variables using CBA [4], implementing a multiple correspondence analysis (MCA) on complete disjunctive tables [5]. The main innovation lies in the integration of plithogenic fuzzy soft sets to explicitly model three key components in categorical data: (1) the certainty in classifications, (2) the ambiguity in category assignments, and (3) the potential contradiction between successive assessments. This hybrid STATIS-CBA-Plithogenic approach allows capturing not only observable changes between occasions, but also the uncertainty inherent in complex measurement processes, such as those found in nutritional studies where multiple biological and social factors interact.

The proposed adaptation retains all the advantages of the original STATIS method while incorporating the ability to: (a) quantify degrees of indeterminacy in individual trajectories, (b) weight associations between variables according to their temporal consistency, and (c) identify patterns of change that would be considered atypical in traditional approaches, but that may reflect genuine transitions when analyzed under this broader perspective.

2. Preliminaries

2.1. Adaptation of STATIS to the CBA

Following Fajardo et al [6], the interstructure stage of the STATIS methodology in this case requires the following definition of the information block for each occasion:

$$E = (Z_k, M_{Xk}, D_{Xk}) \quad (1)$$

where Z_k is a partitioned matrix in which the disjunctive tables of each variable in block k are juxtaposed:

$$Z_k = (Z_{k(1)}, Z_{k(2)}) \quad (2)$$

M_{Zk} is the metric introduced into the category space of the variables:

$$M_{Zk} = \begin{pmatrix} D_{Zk(1)}^{-1} & 0 \\ 0 & D_{Zk(2)}^{-1} \end{pmatrix} \quad (3)$$

where $D_{Zk(q)} = Z_k^t Z_k$ a diagonal matrix containing the frequencies with which the J_q categories of the q-th variable in the study occur in block k, q=1,2; and $D_{Zk} = I_n$, is the weight matrix of the individuals.

Each study is assigned a matrix array called an object, which contains the information required to describe the distances between the individuals in the block:

$$W_k = Z_k M_{Xk} Z_k^t = \left(Z_{k(1)}, Z_{k(2)}\right) \begin{pmatrix} D_{Zk(1)}^{-1} & 0 \\ 0 & D_{Zk(2)}^{-1} \end{pmatrix} \begin{pmatrix} Z_{k(1)}^t \\ Z_{k(2)}^t \end{pmatrix} \quad (4)$$

It is shown that the Hilbert-Schmidt (HS) scalar product of object W_k with itself is a function of the chi2 statistic between the variables:

$$< W_k, W_k > = g_k\left(\chi^2_{xk(1),xk(2)}\right) = (J_1 + J_2 + 2) + (2/n)\, \chi^2_{xk(1),xk(2)} \quad (5)$$

and that the scalar product HS of the objects W_k and W_m is a function of the association between the variables in both blocks:

$$< W_k, W_m > = g_{km} \left(\sum_{s=1}^{2} \sum_{r=1}^{2} \chi^2_{xk(s),xm(r)} \right)$$

$$= (1/n) \left(\chi^2_{xk(1),xm(1)} + \chi^2_{xk(1),xm(2)} + \chi^2_{xk(2),xm(1)} + \chi^2_{xk(2),xm(2)} \right) + 4 \qquad (6)$$

In this way, the distance between objects W_k and W_m will be smaller the greater the association between the variables in both blocks:

$$d^2 (W_k, W_m) = g_k \left(\chi^2_{xk(1),xk(2)} \right) + g_m \left(\chi^2_{xm(1),xm(2)} \right) - 2\, g_{km} \left(\sum_{s=1}^{2} \sum_{r=1}^{2} \chi^2_{xk(s),xm(r)} \right) \qquad (7)$$

To obtain a representation space for the different blocks, a principal component analysis is performed on the matrix S, which contains the scalar products between the objects of the different blocks:

$$S = (< W_k, W_m >) \qquad k, m = 1, 2\ldots K \qquad (8)$$

The principal directions of the representation space in reference are denoted by $\{a^\alpha : \alpha=1,2\ldots K\}$, eigenvectors associated with the respective eigenvalues μ_α ($\alpha=1,2\ldots K$) of the matrix S.

The properties of this representation space are as follows:

- $\sum_{\alpha=1}^{K} \mu_\alpha = \sum_{k=1}^{K} < W_k , W_k > = \sum_{k=1}^{K} g_k(\chi^2_{xk(1),xk(2)})$ is a measure of the overall association between the categorical variables that identify the different occasions. Therefore, the strength of the α-th direction in this space is given by:

  $$\frac{\mu_\alpha}{\sum_{\alpha=1}^{K} \mu_\alpha}$$

  This ratio can be interpreted as the proportion of the overall association between the variables X_1 and X_2, across all blocks, captured by the α-th principal address.

- The length of the vector that connects an object to the origin of the coordinates on the representation planes approximates the function $g_k(\chi^2_{xk(1),xk(2)})$, a measure of the association between the two variables within the block.

- The distance between two objects on the representation planes, as already noted, is an inverse function of the association between the variables that respectively identify the blocks considered.

The summary object, called the compromise object, is defined as a linear combination with certain restrictions on the coefficients:

$$W = \alpha_1 W_1 + \alpha_2 W_2 + \ldots + \alpha_K W_K \qquad (9)$$

where the vector α maximizes the sum of squares of the covariances (in the HS sense), between the compromised object and the objects in the blocks:

$$\max_{\alpha} \sum_{h=1}^{K} \mathrm{Cov}^2 (W, W_h) \leftrightarrow \max_{\alpha} \sum_{h=1}^{K} < W, W_h >^2 \qquad (10)$$

sought coefficient vector is the normalized eigenvector associated with the first eigenvalue of the matrix S.

To obtain the representation of the individuals in the compromise space, the matrix W is written as a Grammian matrix W = AA^t, where A = (A $_{(1)}$, A $_{(2)}$ … A $_{(K)}$), being:

$$A_{(k)} = \sqrt{\alpha_k}\, Z_{(k)_{nxJ}}\; D^{-1/2}_{(k)JxJ} \tag{11}$$

The coordinate vector of the individuals on the -th axis is then αgiven by:

$$\psi^{\alpha} = A_{nx(K*J)}\; V^{\alpha}_{(K*J)x1} \tag{12}$$

So if we assume that on occasion k the i-th individual possesses modalities j1(k) of the first variable and j2(k) of the second variable, its projection coordinate on the α-th axis of the commitment space takes the form:

$$\psi_{i\,\alpha} = \sum_{k=1}^{K} \sqrt{\alpha_k}\; \left(\frac{(k_{j1(k)})^{-1/2}\; v^{\alpha}_{(k)\,j1} + (k_{j2(k)})^{-1/2}\; v^{\alpha}_{(k)\,j2}}{2}\right) \tag{13}$$

which is, except for the factor $\sqrt{\alpha_k}$ the half-sum of the subvector coordinates $V^{\alpha}_{(k)\,Jx1}$weighted, respectively, by the inverse of the square roots of the number of individuals presenting those categories. These weights give greater weight to the less frequent categories, as is generally the case in CBA.

Finally, the representation of individuals' trajectories in the compromise space is achieved by:

$$\psi^{\alpha} = \lambda_{\alpha}^{-1/2}\; W_{nxn}\; U^{\alpha}_{nx1} \tag{14}$$

and if we consider each object W_k as a supplementary arrangement, it can be represented on the compromise space by the expression:

$$\text{Proy}_{U\alpha}(W_k) = \lambda_{\alpha}^{-1/2}\; W_k\; U^{\alpha}_{nx1} \tag{15}$$

2.2. Soft sets and extensions

Definition 1 ([7]). A *smooth set* over $U$is a pair (F, E), where is the Uinitial universal set , $E$is the set of parameters and $F$is the map of $E$to $\mathcal{P}(U)$, which is the power set of U.

So, given a parameter $\varepsilon \in E$, we have $F(\varepsilon) \in \mathcal{P}(U)$as the set of ε −approximate elements of (F, E).

Definition 2 ([7]). A *fuzzy The smooth set* on U is a pair (F, E), where is the initial U universal set , E is the set of parameters and F is the mapping of E to $\mathcal{F}(U)$, which is the set of its bfuzzy Usets .

Definition 3 ([7]). An *intuitionist Diffuse The smooth set* on U is a pair (F, E), where U is the initial universal set, E is the set of parameters and F is the map of E to $\mathcal{IF}(U)$, which is the set of intuitionistic fuzzy subsets of U.

Definition 4 ([7]). A *Neutrosophic The smooth set* over U is a pair (F, E), where U is the initial universal set, E is the parameter set and $F$is the map of E to $\mathcal{N}(U)$, which is the set of neutrosophic subsets of U.

2.3. Plithogenic assemblages and soft plithogenic assemblages

U is the universe of discourse, then fix P, which is a non-empty set of elements, and $P \subset U$[1, 2]. It further follows that A is the non-empty set of *one-dimensional attributes* , such that $A = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$, $m \geq 1$. With each, we have a spectrum of all $\alpha \in A$ possible values (or states) .S which may be a discrete finite set $S = \{s_1, s_2, \dots, s_l\}$, or a $1 \leq l < \infty$ countably $S = \{s_1, s_2, \dots, s_\infty\}$ infinite set, or an uncountably infinite (continuous) set $S =]a, b[, a < b.$] ... [denotes any open, half-open, or closed interval of the set of real numbers or another general set.

On the other hand, $V \subset S$ and $V \neq \emptyset$ is the range of all attributes that experts need for the given application. Then, for each $x \in P$, let $V = \{v_1, v_2, \dots, v_n\}$ be the set of its attribute values, such that $n \geq 1$

There is generally a value called *the dominant attribute value* in V, which is selected by experts based on their criteria as to which is the most important attribute to meet the proposed objective.

The item $v \in V$ has an *approval rating.* $d(x, v)$ from element x, to set P, for some assumed criteria.

The degree of belonging is classified as *the diffuse degree of belonging*, a *diffuse intuitionistic degree of belonging*, or a *neutrosophic degree of belonging* to the plithogenic set.

So, we have the *value of the attribute accessory degree function* as*:*
$\forall x \in P, d: P \times V \rightarrow \mathcal{P}([0,1]^z)$(1)

That is, $d(x, v)$ is a subset of $[0,1]^z$, such that $\mathcal{P}([0,1]^z)$ is the power set of $[0,1]^z$, where determines the z*membership* type. In particular, z = 1 means a *fuzzy degree of belonging*, z = 2 denotes an *intuitionistic fuzzy degree of belonging,* yz = 3 is for the *neutrosophic degree of belonging*.

The function $c: V \times V \rightarrow [0,1]$ is the *degree of contradiction function of the attribute value* between any two attribute values $v_1$and v_2. This satisfies the following axioms:

1. $c(v_1, v_1) = 0$, that is, the degree of contradiction between the same attribute values is zero;

2. $c(v_1, v_2) = c(v_2, v_1)$, commutativity.

There is a distinction between functions caccording to the value z. The *degree of contradiction function of the fuzzy attribute value is denoted by* c_F, the *degree of contradiction function of the intuitionistic fuzzy attribute value* is a function $c_{IF}: V \times V \rightarrow [0,1]^2$, while the *degree of contradiction function of the neutrosophic attribute value is* defined by $c_N: V \times V \rightarrow [0,1]^3$.

Generally, these are one-dimensional attribute values and their degree of discrepancy. If you have multidimensional attribute values, these can be broken down into one-dimensional attribute values.

The attribute value degree of contradiction function allows for greater precision when performing calculations on some grouping methods and ranking systems. These values are based on expert judgment regarding the specific problem to be solved. If *an attribute cannot be determined, the precision of the value degree of contradiction function will be lost, although the entire theory can still be used.*

Once the above concepts have been defined,(P, a, V, d, c) It is a *plithogenic set* that meets the following:

1. P is a set, a is a one-dimensional or generally multidimensional attribute, V is the range of the attribute values, d is the degree of membership of the attribute value of each element $x$to the set P, $x \in P$, for some given criteria. Finally, d is d_F, d_{IF}, or d_N, when it is a fuzzy degree of membership, an intuitionistic fuzzy degree of membership or a neutrosophic degree of membership, respectively, of an element x to the plithogenic set P;

2. On the other hand, we define c as c_F, c_{IF} or c_N, if it is the fuzzy degree of contradiction, intuitionistic fuzzy degree of contradiction or neutrosophic degree of contradiction between attribute values, respectively.

Experts define $d(\cdot,\cdot)$ and $c(\cdot,\cdot)$ define the domain of specialization in which they operate.

The notation used is as follows:

$x(d(x,V))$, where $d(x,V) = \{d(x,v),\ for\ all\ v \in V\}, \forall x \in P$.

To calculate the degree of contradiction of the attribute value, it is performed on each value of the particular attribute and the value of the dominant attribute, called v_D.

The function of degree of contradiction of attribute value c among attribute values is included in the definition of *plithogenic aggregation operators* (intersection (AND), union (OR), implication (⟹), equivalence (⟺), inclusion relation (partial order) and other plithogenic aggregation operators that combine two or more degrees of attribute value acting on t-norm and t-conorm.

Most plithogenic aggregation operators are linear combinations of the fuzzy t-norm ($\wedge_F$) and the fuzzy t-conorm ($\vee_F$). Nonlinear combinations can also be defined.

Having the calculation of the t-norm and the t- conorm between the dominant attribute value (v_D) with another attribute value (v_2), and also $c(v_D, v_2)$ denotes the contradiction between v_D and v_2, then we can define the following operations:

$$[1 - c(v_D,v_2)] \cdot t_{norm}(v_D,v_2) + c(v_D,v_2) \cdot t_{conorm}(v_D,v_2), \quad (16)$$

or what is the same:

$$[1 - c(v_D,v_2)] \cdot (v_D \wedge_F v_2) + c(v_D,v_2) \cdot (v_D \vee_F v_2), \quad (17)$$

Also,

$$[1 - c(v_D,v_2)] \cdot t_{conorm}(v_D,v_2) + c(v_D,v_2) \cdot t_{norm}(v_D,v_2), \quad (18)$$

either,

$$[1 - c(v_D,v_2)] \cdot (v_D \vee_F v_2) + c(v_D,v_2) \cdot (v_D \wedge_F v_2). \quad (19)$$

The *plithogenic neutrosophic intersection* is defined in equation (20):

$$(a_1,a_2,a_3) \wedge_P (b_1,b_2,b_3) = \left(a_1 \wedge_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \vee_F b_3\right), \quad (20)$$

The p*lithogenic neutrosophic union* is as follows:

$$(a_1,a_2,a_3) \vee_P (b_1,b_2,b_3) = \left(a_1 \vee_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \wedge_F b_3\right), \quad (21)$$

To define the p*lithogenic neutrosophic inclusion* we have:
Since the degrees of contradiction are $c(a_1,a_2) = c(a_2,a_3) = c(b_1,b_2) = c(b_2,b_3) = 0.5$, then: $a_2 \geq [1 - c(a_1,a_2)]b_2$ or $a_2 \geq (1-0.5)b_2$ or $a_2 \geq 0.5b_2$ and $c(a_1,a_3) = c(b_1,b_3) = 1$.
When $a_1 \leq b_1$ the converse applies to $a_3 \geq b_3$, and then $(a_1,a_2,a_3) \leq_P (b_1,b_2,b_3)$ if and only if $a_1 \leq b_1$ and $a_2 \geq 0.5b_2$, $a_3 \geq b_3$.

Applications of plithogenic sets and plithogenic logic can be read in [8-12].

Definition 5 ([13,14]). Let be U a universe of discourse, $\mathcal{P}([0,1]^z)$ is the power z of U, such that:

- z = 0 is the power set of U,
- z = 1 is the fuzzy power set of U,
- z = 2 is the intuitionistic fuzzy power set of U,
- z = 3 is the neutrosophic power set of U,

Let be $\alpha_1, \alpha_2, \ldots, \alpha_m$, $m \geq 1$, m different attributes, whose attribute values lie in the sets $V_1, V_2, \ldots, V_m$, such that $V_i \cap V_j = \emptyset$ if $i \neq j$, and $i,j \in \{1,2,\ldots,m\}$. Suppose that $V_i = \left\{v_{i_1}, v_{i_2}, \ldots, v_{i_{n_i}}\right\}$ and also $\Upsilon = V_1 \times V_2 \times \ldots \times V_m$. $D = \left\{v_{D_1}, v_{D_2} \ldots, v_{D_m}\right\}$ are the dominant attribute elements of A_i, and $c_i(v_{D_i}, v_{i_j})$ is the attribute contradiction degree function such that: $c_i: V_i \times V_i \to [0,1]$. We say that the pair (F_P^z, Υ) is the *Plithogenic Smooth Set* (PSS) over U, such that:

$$F_P^z: \Upsilon \to [0,1]_D \times \mathcal{P}([0,1]^z) \quad (22)$$

Definition 6 ([13,14]). The union of two PSSs (F_P^z, A) and (G_P^z, B)over U, denoted by $(F_P^z, A) \vee_P^z (G_P^z, B)$ is the PSS (H_P^z, Ω), where $\Omega = A \cup B$ such that $\forall \varepsilon \in \Omega$,

$$H_P^z(\varepsilon) = \begin{cases} F_P^z(\varepsilon), if\ \varepsilon \in A \setminus B \\ G_P^z(\varepsilon), if\ \varepsilon \in B \setminus A \\ F_P^z(\varepsilon) \vee_P^z G_P^z(\varepsilon), if\ \varepsilon \in B \cap A \end{cases}$$

where $\vee_P^z$ is the z- *plithogenic junction.*

Definition 7 ([13,14]). The intersection of two PSSs $(F_P^z, A)$and (G_P^z, B) on U, denoted by $(F_P^z, A) \wedge_P^z (G_P^z, B)$ is the PSS (H_P^z, Ω), where $\Omega = A \cap B$ such that $\forall \varepsilon \in \Omega$,

$$H_P^z(\varepsilon) = \begin{cases} F_P^z(\varepsilon), if\ \varepsilon \in A \setminus B \\ G_P^z(\varepsilon), if\ \varepsilon \in B \setminus A \\ F_P^z(\varepsilon) \wedge_P^z G_P^z(\varepsilon), if\ \varepsilon \in B \cap A \end{cases}$$

where $\vee_P^z$ is the z- *plithogenic intersection.*

Definition 8 ([13,14]). Given $(F_P^z, E)$and $(G_P^z, E)$are two probability scoring systems (PSS) over (U, E). The similarity between (F_P^z, E) and (G_P^z, E) is denoted by $\mathcal{S}(F_P^z, G_P^z)$and is defined by:

$$\mathcal{S}(F_P^z, G_P^z) = \frac{1}{|E|} \sum_{k=1}^{|E|} M_k \qquad (23)$$

$$M_k = 1 - \frac{\sum_{j=1}^{|U|} \sum_{i=1}^{|e|} |F_j(e_{ik}) - G_j(e_{ik})|}{\sum_{j=1}^{|U|} \sum_{i=1}^{|e|} |F_j(e_{ik}) + G_j(e_{ik})|}, \text{ for } e \in E.$$

Definition 9 ([13]). Given $(F_P^z, E)$and $(G_P^z, E)$are two probability scoring systems (PSS) over (U, E). We say that $(F_P^z, E)$and $(G_P^z, E)$are *significantly similar* if $\mathcal{S}(F_P^z, G_P^z) \geq \frac{1}{2}$.

Properties: Given (F_P^z, E), (G_P^z, E), and (H_P^z, E), are three PSS over (U, E), then:

(1) $\mathcal{S}(F_P^z, G_P^z) = \mathcal{S}(G_P^z, F_P^z)$,
(2) $0 \leq \mathcal{S}(F_P^z, G_P^z) \leq 1$,
(3) $F_P^z = G_P^z$implies $\mathcal{S}(F_P^z, G_P^z) = 1$.
(4) $F_P^z \subseteq G_P^z \subseteq H_P^z$implies $\mathcal{S}(F_P^z, H_P^z) \leq \mathcal{S}(G_P^z, H_P^z)$.

3. An application

In order to validate the proposed method, it has been applied to a follow-up study in which the nutritional status of 66 schoolchildren between 6 and 8 years old was evaluated. Two interannual measurements were taken between 2017 and 2018, as part of a project that included, among other aspects, food supplementation (breakfast, lunch, and dairy drink) [15]. Non-invasive anthropometric indicators were used to assess nutritional status: 1) Height for age, a key measurement to detect growth retardation, also known as chronic malnutrition; and 2) Body Mass Index (BMI), which has been considered a reliable indicator to detect obesity, overweight, or weight deficiency in relation to height. The categorization of both indicators, into "below the norm," "normal," and "above the norm," with reference to the pattern proposed by the World Health Organization [16], have been denoted here as TE and PT, respectively.

Analysis of object W_{2017}

Analysis of the information contained in object W_{2017} using CBA indicates the existence of two main directions, which capture more or less similar proportions of the association between the TE and PT indicators. The representation on the factorial plane has allowed us to recognize three groups of children: 1) Children with an adequate nutritional status both by TE and PT (67.0%), 2) Children with global malnutrition and absence of chronic malnutrition (16.7%), and 3) Children with chronic malnutrition and absence of global malnutrition (16.7%) (Figure 1).

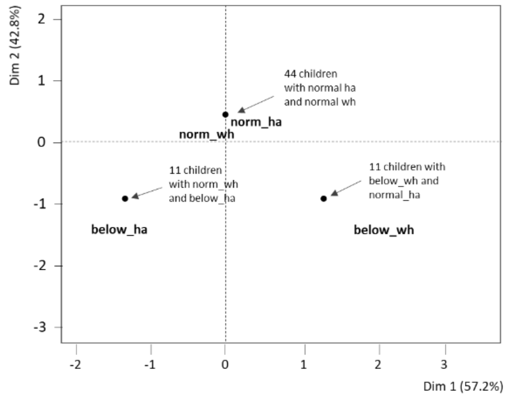

Figure 1. Representation of children and categories 2017

Analysis of object W_{2018}

The CBA applied to object W_{2018}, although leading to essentially similar results as in 2017 in terms of the main directions, reflects important changes in the positions of individuals, as shown in Figure 2, compared to the analogous figure in 2017. 1) Of the 16 children who in 2018 present ET below the norm and PT normal, 10 had this same condition in 2017, 4 had been classified as normal in both indicators in 2017 and 2 had normal ET and PT below the norm; 2) Of the 5 children who in 2018 were diagnosed with normal ET and PT under the norm, 3 presented the same condition in 2017 and 2 deteriorated only in relation to PT; 3) Of the 45 children who exhibited normal nutritional status in 2018, both by TE and PT, 39 had a similar diagnosis in 2017 and 6 had a diagnosis of global malnutrition in 2017, indicating that the nutritional supplement had an effect on them.

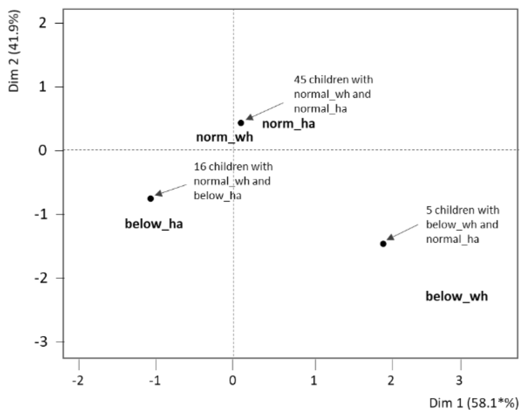

Figure 2. Representation of children and categories 2018.

Interstructure

The analysis of the interstructure (Figure 3) reveals the similarities between the objects W_{2017} and W_{2018}, which is highlighted in their projections on the first axis, which has the greatest explanatory power (88%) of the association between the variables at the two moments, also reflecting differences of a lower degree captured by the second direction (12%). The projection of the commitment object (W_c) shows that it adequately represents the objects at the two moments under study, as reflected both by the magnitude of the coefficients that define it, as well as the RV coefficients [17], of the commitment to the two objects under consideration ($RV(W_c, W_{2017}) = RV(W_c, W_{2018}) = 0.938$). For its part, the coefficient $RV(W_{2017}, W_{2018}) = 0.76$, allows us to point out that there is a close relationship between the previous nutritional evaluation, mediated by a food supplementation and the evaluation carried out later.

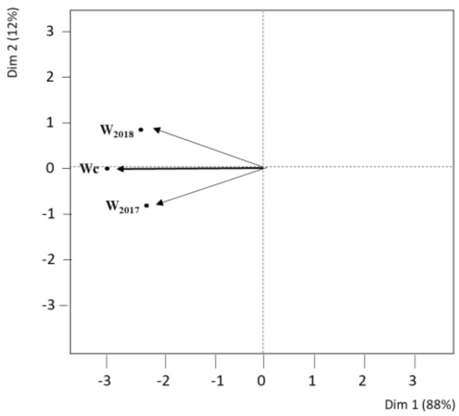

Figure 3. Representation of W_{2017} and W_{2018} on compromise space

Infrastructure

At this stage of the analysis, it is possible to explain the similarities and/or differences between objects W_{2017} and W_{2018} by interpreting the axes of the commitment space, taking into account the projection coordinates of the categories of the variables TE and PT, which accounts for the prevalence of the individuals in whom they are present (Figure 4). It can be noted that the first axis of the space captures 46.1% of the association between these variables. It was found that, as in the separate analyses of the two instants, the first axis of the commitment establishes an opposition between children with global malnutrition and children diagnosed with chronic malnutrition, and the second axis, which captures 29.7%, contrasts children with a normal nutritional evaluation with respect to the two groups mentioned above. In a certain sense, the close association observed in the measurements of the same variable on both occasions justifies the fact that they appear located in projection around the same point.

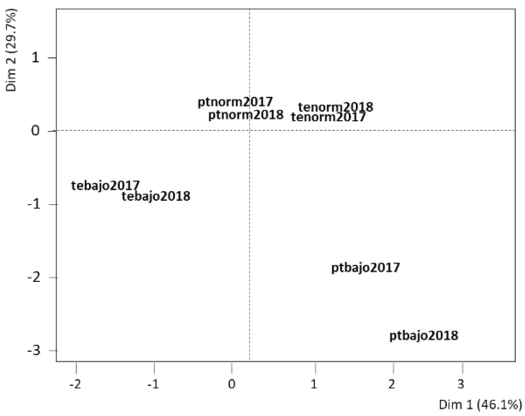

Figure 4. Representation of categories on the compromise space.

The representation of children on the commitment space describes differences between them determined by the changes that occur in their nutritional status between 2017 and 2018, mediated by the supplementary feeding diet, their projections on the space, follow the outline of the parabola defined by a Guttman effect, suggesting the following typology, according to the changes observed in TE and PT, between periods (Figure 5). It is important to note that the food and supplement that children receive at school only guarantees them part of the daily requirement, which must be completed with the contribution of meals at home, of which we are not certain, due to the socioeconomic conditions of these families.

Type 1 Children (1212). Represented on the third quadrant of the plane, with a diagnosis of chronic malnutrition in 2017 and a condition of global normality, a nutritional profile that remains in 2018. The supplementary food diet had no effect on this group of children (n = 10), which may be due to the fact that the recovery of height requires, in addition to an adequate and sustained diet that covers their caloric requirements, the control of other intervening environmental and affective factors that persist over time.

Type 2 Children (1222). Exactly along axis 1, their nutritional profile indicates a pattern of change that removes them from chronic malnutrition from 2017 to 2018, maintaining their normal PT condition in both periods (n=1). These changes are possibly more evident when the level of height impairment is mild and the intervening factors can be attenuated in the short term.

Type 3 Children (2212). Also defined by the direction of axis 1, with a change pattern that defines an involution from normal TE in 2017 to chronic malnutrition in 2018, maintaining their normal PT between the two measurement times (n=3). These children would possibly be very close to the lower limit of TE (-2ZT), and therefore very vulnerable to any negative event in a given time period.

Type 4 Children (2222). Located towards the top of the plane, they showed a normal nutritional profile in both measurements (n=44). Interestingly, a pattern of normality was identified across both measurements, which suggests that in this region there are protective factors, in addition to nutrition, that allow most children to grow along appropriate paths for their age.

Type 5 Children (2122). Projected towards the bottom of the plane, in the positive direction of the first axis, with a nutritional profile that in the initial assessment indicates global malnutrition with absence of chronic malnutrition, children who responded to the supplementary feeding diet, recovering a normal nutritional status both by TE and PT in 2018 (n = 6). In this group of children, the recovery is possibly due to the fact that they have gained weight, in response to the food and supplementation received, to the family diet and to the control of the intervening environmental and home factors.

Type 6 children (2221). Positioned towards the bottom of the fourth quadrant, characterized by a normal nutritional profile in 2017, mutating towards global malnutrition in 2018, although they maintain their normal ET condition (n=2). Possibly due to other environmental factors, such as deterioration in public services, drinking water, solid waste or infections, which have increased requirements and limited weight gain, but have not been sufficiently intense or prolonged to affect height.

Type 7 Children (2112). Also arranged in the lower area of the plane, between the third and fourth quadrants, with normal TE and PT below the norm at the beginning, reversing the pattern at the end of the study, with TE below the norm and normal PT (n=2). They have recovered the PT, but this increase in weight has not been sufficient for it to be accompanied by the increase in height. It may also indicate that the increase in weight while maintaining the level of growth retardation in height, achieves that the PT is located as adequate.

Type 8 Children (2121). Represented in the lower right region of the plane, with a diagnosis of absence of chronic malnutrition, but with global malnutrition in 2017, a profile that remains the same in 2018. The supplementary food diet had no effect on this group of children (n=3).

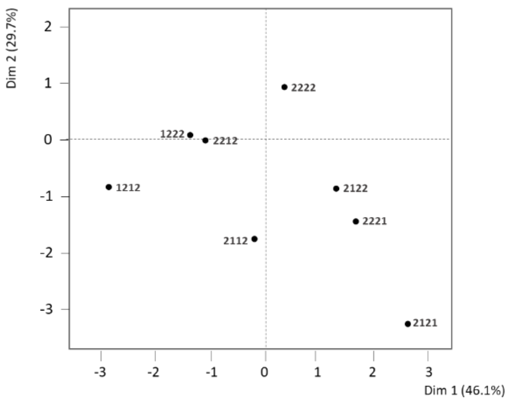

Figure 5. Representation of children on compromise space.

Trajectories

The tracing of trajectories allows a more precise description of changes in children's nutritional status before and after the application of the nutritional supplement. Large trajectories generally describe a significant change in nutritional status from 2017 to 2018, while shorter trajectories show little or no interannual change. The following graphs present the trajectories of selected children belonging to some of the typologies mentioned above. Figure 6a illustrates three short trajectories indicating that the nutritional profile is maintained between assessments. Figure 6b shows two long trajectories, both directed toward the low ET condition. Figure 6c presents two trajectories oriented toward the adequate PT condition.

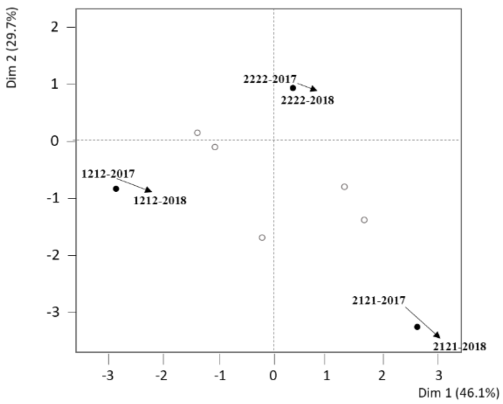

Figure 6a. Representation of unchanged children on compromise space. Types 1212, 2121 and 2222.

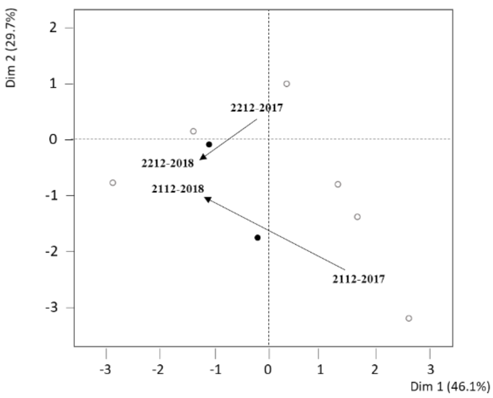

Figure 6b. Representation of children on compromise space. Types 2212 y 2112.

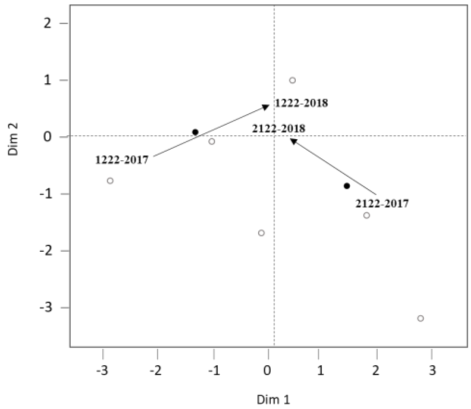

Figure 6c.- Representation of children on compromise space. Types 2122 y 1222

3.1. Plithogenic Analysis of Nutritional Trajectories

To complement the STATIS-ACB analysis, the **Plithogenic Fuzzy Soft Set Method**, based on the plithogenic theory described by Smarandache [12], is applied to model the uncertainty and ambiguity in the nutritional trajectories of the 66 schoolchildren assessed in 2017 and 2018. This approach assigns to each combination of anthropometric categories (Height-for-Age, TE, and Body Mass Index, PT) a set of neutrosophic values (truth, T; indeterminacy, I; falsity, F) in [0,1], capturing the certainty, uncertainty, and contradiction in the associations between the "below the norm," "normal," and "above the norm" categories. The analysis focuses on the typologies identified in Figure 5, using the frequencies and transitions reported in the W_{2017} and W_{2018} object analysis sections.

3.1.1. Applied Plithogenic Methodology

A soft plithogenic set is defined $\boldsymbol{P} = (\boldsymbol{U}, \boldsymbol{A}, \boldsymbol{V}, \boldsymbol{f}, \boldsymbol{c})$, where:

- **U** is the universe of the 66 schoolchildren
- $\boldsymbol{A} = \{\mathbf{2017}, \mathbf{2018}\}$ is the set of parameters (years of evaluation)
- **V** is the range of attribute values, represented by the combinations of categories: $\boldsymbol{V} = \{\boldsymbol{TE\ bajo/PT\ bajo}, \boldsymbol{TE\ bajo/PT\ normal}, \boldsymbol{TE\ normal/PT\ bajo}, \boldsymbol{TE\ normal/PT\ normal}\}$ denoted as v_1, v_2, v_3, v_4
- $\boldsymbol{f}: \boldsymbol{U} \times \boldsymbol{A} \rightarrow \boldsymbol{Pz}(\boldsymbol{V})$ is the plithogenic membership function, with $z = 3$ (neutrosophic membership), which assigns to each student in each year a vector (T, I, F) for each combination
- $\boldsymbol{c}: \boldsymbol{V} \times \boldsymbol{V} \rightarrow [\mathbf{0}, \mathbf{1}]$ is the degree of contradiction function, defined based on the dissimilarity between categories.

Contradiction Function

The dominant attribute is set as v_4 (normal TE/normal PT), as it represents the optimal nutritional status according to the WHO [8]. The contradiction function $c(v_i, v_4)$ is defined according to the semantic distance between categories, based on the nutritional severity:

- $c(v_1, v_4) = 0.8$ (Low TE/low PT vs. normal TE/normal PT, high contradiction due to severe malnutrition)
- $c(v_2, v_4) = 0.6$ (Low TE/normal PT, moderate contradiction due to chronic malnutrition)
- $c(v_3, v_4) = 0.6$ (Normal TE/low PT, moderate contradiction due to global malnutrition)
- $c(v_4, v_4) = 0$ (same category, no contradiction)

Assignment of Neutrosophic Values

For each combination, values are assigned (T, I, F)based on the reported frequencies (Figures 1 and 2). For example:

- For v_4 (normal TE/normal PT) in 2017 (67%, 44 children): $\boldsymbol{f(v_4, 2017) = (0.7, 0.2, 0.1)}$where T = 0.7 reflects the high prevalence, I = 0.2 the uncertainty due to external factors (e.g. , family diet), and F = 0.1 the low probability of non-membership.
- For v_2 (low TE/normal PT) in 2018 (16 children): with I = 0.3 due to the uncertainty in the persistence of $\boldsymbol{f(v_2, 2018) = (0.6, 0.3, 0.1)}$chronic malnutrition .

Plithogenic Aggregation

Plithogenic aggregation is performed using the plithogenic neutrosophic intersection:

$$\boldsymbol{P_1 \wedge P_2 = (T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2) \cdot (1 - c(v_i, v_j))}$$

Where ∧ and ∨ are the fuzzy t-norm (minimum) and t- conorm (maximum), and c(v_i , v_j) weights the contradiction .

The overall plithogenic index for each combination is calculated as:

$$\boldsymbol{\acute{I}ndice = T/(T + I + F)}$$

3.1.2. Plithogenic Results

Table 1. Neutrosophic Values and Plithogenic Indices for Category Combinations

Combination	Year	T	Yo	F	Plithogenic Index (Corrected)
Normal TE/Normal PT (v_4)	2017	0.70	0.20	0.10	**0.70**
Normal TE/Normal PT (v_4)	2018	0.75	0.20	0.05	**0.75**
Low TE/Normal PT (v_2)	2017	0.60	0.30	0.10	**0.60**
Low TE/Normal PT (v_2)	2018	0.60	0.30	0.10	**0.60**
Normal TE/Low PT (v_3)	2017	0.50	0.30	0.20	**0.50**
Normal TE/Low PT (v_3)	2018	0.50	0.35	0.15	**0.50**
Low TE/Low PT (v_1)	2017	0.40	0.40	0.20	**0.40**
Low TE/Low PT (v_1)	2018	0.40	0.40	0.20	**0.40**

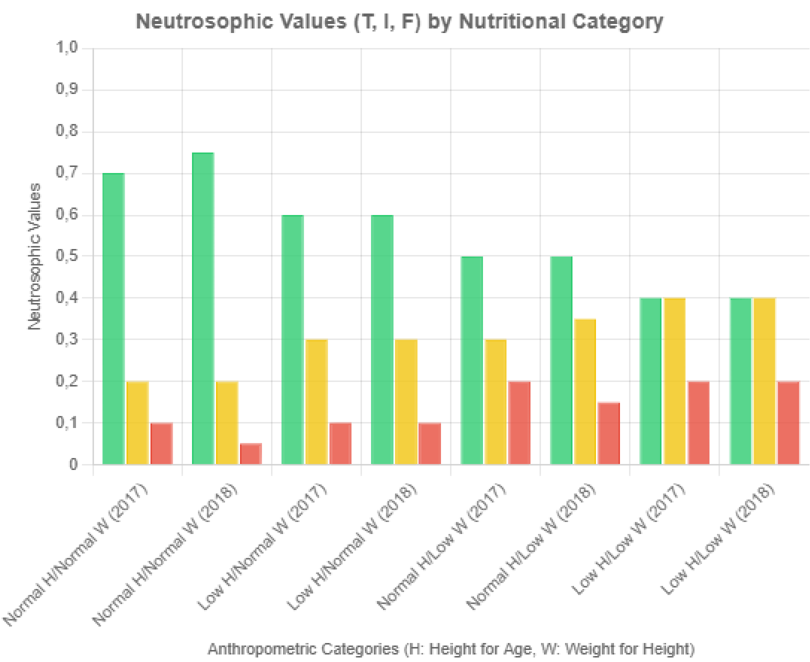

Figure 7. Plitogenic Analysis of Nutritional Trajectories in Schoolchildren.

Table 2. Analysis of Trajectories by Typology

Typology	Description	Number of Children	Index 2017 (Corrected)	Index 2018 (Corrected)	Interpretation
Type 4 (2222)	Normal TE/Normal PT → Normal TE/Normal PT	44	**0.70**	**0.75**	Optimal nutritional stability with slight improvement
Type 5 (2122)	Low TE/Normal PT → Normal TE/Normal PT	6	**0.60**	**0.75**	Significant improvement post-supplementation
Type 1 (1212)	Low TE/Normal PT (persistent)	10	**0.60**	**0.60**	Stable chronic malnutrition

Analysis of Key Transitions

- **Type 4 (normal TE/normal PT).** The 44 children in this category show a plithogenic index that increases from **0.70 to 0.75**. This confirms the **stability and slight improvement** in optimal nutritional status, although an indeterminacy (I=0.2) remains due to external factors.
- **Type 5 (Low TE/Normal PT → Normal TE/Normal PT)**: The 6 children who transitioned to an optimal state show a change in the plithogenic index from **0.60 in 2017 to 0.75 in 2018.** This still indicates a **very significant nutritional improvement**, reinforcing the interpretation of the positive impact of supplementation.

- **Type 1 (Low TE/Normal PT)**: The 10 children in this category maintain a stable index of **0.60**. The value of T = 0.6 and the high indeterminacy of I = 0.3 correctly reflect the **persistence of chronic malnutrition**.

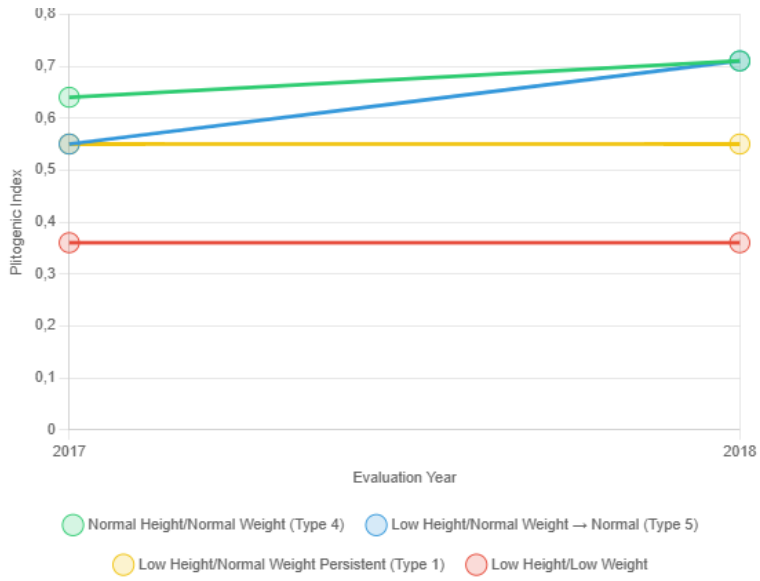

Figure 8. Temporal evolution of plithogenic indices according to nutritional trajectory typologies.

Aggregation Calculations

The aggregation calculations presented in the original document **are correct** and do not require modification.

- Aggregation for v_4 (2017 and 2018):
  $$f(v_4, 2017) \wedge f(v_4, 2018) = (min(0.7, 0.75), max(0.2, 0.2), max(0.1, 0.05)) \cdot (1 - 0) = (0.7, 0.2, 0.1).$$

- Aggregation for Type 5 transition(v_2 *en* 2017 $\rightarrow$ v_4 *en* 2018):
  $$f(v_2, 2017) \wedge f(v_4, 2018) = (min(0.6, 0.75), max(0.3, 0.2), max(0.1, 0.05)) \cdot (1 - 0.6) = (0.6, 0.3, 0.1) \cdot 0.4 = (0.24, 0.12, 0.04).$$
  - Aggregate plitogenic index : $0.24 / (0.24 + 0.12 + 0.04) = 0.24 / 0.40 = 0.60.$

3.1.3. Corrected Plithogenic Discussion

The plithogenic analysis, with the corrected numerical indices, **reinforces and validates the trajectories** identified by the STATIS-ACB method. The quantitative results confirm the **stability of the optimal nutritional category** (normal TE/normal PT) and the **significant improvement** in Type 5 children, who transitioned from a state of malnutrition to normal nutrition.

A key finding that persists is the evidence of **persistent indeterminacy (I ≈ 0.2–0.3)** across categories. This uncertainty, attributed to uncontrolled factors such as family diet and socioeconomic conditions, demonstrates the advantage of the plithogenic approach for modeling complex realities. Representing trajectories in a plithogenic space offers a more uncertainty-sensitive view, complementing factor analysis and providing a more robust diagnostic tool.

Table 3. Summary of Contradiction Indices by Category.
This table summarizes the values of the contradiction function $c(v_i, v_4)$, which are a methodological component defined to weight the dissimilarity between nutritional categories in aggregation calculations.

Combination of Categories	Contradiction Function $c(v_i, v_4)$	Nutritional Interpretation
Low TE/Low PT → Normal TE/Normal PT	0.8	High contradiction - Severe malnutrition
Low TE/Normal PT → Normal TE/Normal PT	0.6	Moderate contradiction - Chronic malnutrition
Normal TE/Low PT → Normal TE/Normal PT	0.6	Moderate contradiction - Global malnutrition
Normal TE/Normal PT → Normal TE/Normal PT	0.0	No contradiction - Optimal state

Methodological Explanation and Connection with Neutrosophy

- **Link to Neutrosophy**: The analysis uses Plithogenic Fuzzy Soft Sets to model nutritional trajectories using neutrosophic values (T, I, F). This approach simultaneously captures certainty of category membership (T), uncertainty due to external factors (I), and data inconsistency (F), aligning with the foundations of neutrosophic theory.

- **Use of data and results**: The study assigns values (T, I, F) based on the frequencies and transitions reported in previous analyses (Figures 1, 2, and 5). From these values, plithogenic indices are calculated, ensuring that the numerical results are consistent with the input data and the theoretical definitions of the method.

- **Tables and calculations**: The tables and formulas used are based directly on theoretical definitions of plithogeny. Aggregation calculations, which incorporate the contradiction function, and the calculation of the plithogenic index (Index = T / (T + I + F)) are the pillars of the quantitative analysis.

- **Integration with STATIS-ACB**: Plithogenic analysis does not replace, but rather **complements and enhances** STATIS-ACB factor analysis. Its main contribution is to highlight and quantify the uncertainty that traditional factor methods do not explicitly capture, thus offering a more complete diagnosis.

4. Conclusions

The proposed procedure, which adapts the STATIS methodology to correspondence analysis (CBA) and is complemented by Plithogenic Fuzzy Soft Set analysis, is designed to assess changes in phenomena characterized by categorical variables across different occasions, with a particular focus on uncertainty management. Applied to a nutritional assessment study of 66 schoolchildren between 2017 and 2018, the STATIS-CBA method identified trajectories of change in the anthropometric indicators Height-for-Age (HGA) and Body Mass Index (BMI), classified into "below the norm," "normal," and "above the norm." The results revealed key patterns, such as stability in nutritional normality (Type 4, 44 children), improvements in children with initial global malnutrition (Type 5, 6 children), and persistence of chronic malnutrition in others (Type 1, 10 children), reflecting the partial impact of dietary supplementation.

The incorporation of the Plithogenic Fuzzy Soft Set approach enriched the analysis by explicitly modeling the uncertainty inherent in external factors, such as family diet and socioeconomic conditions. This method assigned neutrosophic values (truth, T; indeterminacy, I; falsity, F) to the category combinations. The recalculated plithogenic indices confirm with greater numerical precision the stability of the normal TE/normal PT category (with an index that increased from 0.70 in 2017 to 0.75 in 2018) and the significant improvement in Type 5 (whose index increased from 0.60 to 0.75), but also highlighting a persistent indeterminacy (I ≈ 0.2–0.3) in all trajectories due to uncontrolled factors. This plithogenic perspective complements the STATIS-ACB commitment space, providing a more robust representation of nutritional trajectories by capturing ambiguity and contradiction in the data.

Overall, the hybrid STATIS-CBA-Plithogenic approach facilitates a more accurate diagnosis of nutritional problems at early ages, integrating the statistical rigor of factor analysis with the flexibility of neutrosophy to manage uncertainty. This method has the potential to be applied in other contexts with categorical data subject to temporal variability and external factors, such as epidemiological or social studies, where uncertainty plays a critical role.

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